3/22/17
"The most difficult thing is the decision to act, the rest is merely tenacity."-Emelia Earhart

HW: "Logarithm Laws" Homework section
Test 3 on Thursday 3/30

AIM: What are the Laws of Logarithms?

Warm Up:

1) The expression $\left(\dfrac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to

1) $-\sqrt[6]{m^5}$

2) $\dfrac{1}{\sqrt[6]{m^5}}$

3) $-m\sqrt[5]{m}$

4) $\dfrac{1}{m\sqrt[5]{m}}$

$$= \frac{m^{-1}}{m^{-1/6}} = \frac{1}{m^{5/6}} = \frac{1}{\sqrt[6]{m^5}}$$

switch x and y

2) What is the inverse of the function $y = \log_3 x$?

1) $y = x^3$

2) $y = \log_x 3$

3) $y = 3^x$

4) $x = 3^y$

$x = \log_3 y$

$\downarrow$

$3^x = y$

1. Which of the following is equivalent to $y=\log_7 x$?

(1) $y=x^7$ (3) $x=7^y$

(2) $x=y^7$ (4) $y=x^{1/7}$

Remember, a logarithm "gives" as the y-value the exponent we must raise the base, 7, to in order to get x.

$\boxed{(3)}$

2. If the graph of $y=6^x$ is reflected across the line $y=x$ then the resulting curve has an equation of

~~(1) $y=-6^x$~~ $x=6^y$ (3) $x=\log_6 y$

(2) $y=\log_6 x$ ~~(4) $x=y^6$~~

This question is testing to see if you know that reflecting across the line $y=x$ always produces the inverse, which in this case is $y=\log_6 x$.

$\boxed{(2)}$

3. The value of $\log_5 167$ is closest to which of the following? Hint – guess and check the answers.

(1) 2.67 (3) 4.58

(2) 1.98 (4) 3.18

$5^{3.18}=167.003...\approx 167$

$\boxed{(4)}$

4. Which of the following represents the y-intercept of the function $y=\log(x+1000)-8$?

(1) -8 (3) 3

(2) -5 (4) 5

$y=\log(0+1000)-8=\log(1000)-8$

$=\log_{10}(1000)-8=3-8=-5$

$\boxed{(2)}$

5. Determine the value for each of the following logarithms. (Easy)

(a) $\log_2 32$ (b) $\log_7 49$ (c) $\log_3 6561$ (d) $\log_4 1024$

$\log_2 32=5$ Because $2^5=32$	$\log_7 49=2$ Because $7^2=49$	$\log_3 6561=8$ Because $3^8=6561$	$\log_4 1024=5$ Because $4^5=1024$

6. Determine the value for each of the following logarithms. (Medium)

(a) $\log_2\left(\frac{1}{64}\right)$ (b) $\log_3(1)$ (c) $\log_5\left(\frac{1}{25}\right)$ (d) $\log_7\left(\frac{1}{343}\right)$

$\log_2\left(\frac{1}{64}\right)=-6$ Because $2^{-6}=\frac{1}{2^6}=\frac{1}{64}$	$\log_3(1)=0$ Because $3^0=1$	$\log_5\left(\frac{1}{25}\right)=-2$ Because $5^{-2}=\frac{1}{5^2}=\frac{1}{25}$	$\log_7\left(\frac{1}{343}\right)=-3$ Because $7^{-3}=\frac{1}{7^3}=\frac{1}{343}$

7. Determine the value for each of the following logarithms. Each of these will have non-integer, fractional answers. (Difficult)

(a) $\log_4 2$ (b) $\log_4 8$ (c) $\log_5\sqrt[3]{5}$ (d) $\log_2\sqrt[5]{4}$

$\log_4 2=\frac{1}{2}$ Because $4^{1/2}=\sqrt{4}=2$	$\log_4 8=\frac{3}{2}$ Because $4^{3/2}=\left(\sqrt{4}\right)^3=8$	$\log_5\sqrt[3]{5}=\frac{1}{3}$ Because $5^{1/3}=\sqrt[3]{5}$	$\log_2\sqrt[5]{4}=\frac{2}{5}$ Because $2^{2/5}=\sqrt[5]{2^2}=\sqrt[5]{4}$

EXPONENT AND LOGARITHM LAWS

LAW	EXPONENT VERSION	LOGARITHM VERSION
Product	$b^x \cdot b^y = b^{x+y}$	$\log_b(x \cdot y) = \log_b x + \log_b y$
Quotient	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
Power	$(b^x)^y = b^{x \cdot y}$	$\log_b(x^y) = y \cdot \log_b x$

split

Exercise #1: Which of the following is equal to $\log_3(9x)$?

(1) $\log_3 2 + \log_3 x$ (3) $2 + \log_3 x$

(2) $2\log_3 x$ (4) $x + \log_3 2$

$\log_3 9 + \log_3 x$

$2 + \log_3 x$

Exercise #2: The expression $\log\left(x^2/1000\right)$ can be written in equivalent form as

(1) $2\log x - 3$ (3) $2\log x - 6$

(2) $\log 2x - 3$ (4) $\log 2x - 6$

$\log x^2 - \log 1000$

$2 \cdot \log x - 3$

Exercise #3: If $a = \log 3$ and $b = \log 2$ then which of the following correctly expresses the value of $\log 12$ in terms of a and b?

(1) $a^2 + b$ (3) $2a + b$

(2) $a + b^2$ (4) $a + 2b$

$12 = 2 \cdot 2 \cdot 3$

$12 = 2^2 \cdot 3$

$\log(2^2 \cdot 3)$

$\log 2^2 + \log 3$

$2 \cdot \log 2 + \log 3$

$2b + a$

Exercise #4: Which of the following is equivalent to $\log_2\left(\sqrt{x}/y^5\right)$?

$\sqrt{x} = x^{1/2}$

(1) $\sqrt{\log_2 x} - 5\log_2 y$ (3) $\frac{1}{2}\log_2 x - 5\log_2 y$

(2) $2\log_2 x + 5\log_2 y$ (4) $2\log_2 x - 5\log_2 y$

$\log_2 \sqrt{x} - \log_2 y^5$

$\log_2 x^{1/2} - \log_2 y^5$

$\frac{1}{2}\log_2 x - 5\log_2 y$

Exercise #5: The value of $\log_3\left(\sqrt{5}/27\right)$ is equal to

(1) $\dfrac{\log_3 5-6}{2}$ (3) $\dfrac{\log_3 5-3}{2}$

(2) $2\log_3 5+3$ (4) $2\log_3 5-3$

$\log_3\sqrt{5}-\log_3 27$

$\log_3 5^{1/2} - 3$

$\frac{1}{2}\log_3 5-3$

$\dfrac{\log_3 5}{2}-3$

$\dfrac{\log_3 5}{2}-\dfrac{6}{2}$

$\dfrac{\log_3 5-6}{2}$

Exercise #6: If $f(x)=\log(x)$ and $g(x)=100x^3$ then $f(g(x))=$

(1) $100\log x$ (3) $300\log x$

(2) $6+\log x$ (4) $2+3\log x$

$\log(100x^3)$

$\log 100+\log x^3$

$2+3\log x$

Exercise #7: The logarithmic expression $\log_2\sqrt{32x^7}$ can be rewritten as

(1) $\sqrt{\log_2 35x}$ (3) $\sqrt{5+7\log_2 x}$

(2) $\dfrac{5+7\log_2 x}{2}$ (4) $\dfrac{35+\log_2 x}{2}$

$\log_2\left((32x^7)^{1/2}\right)$

$\frac{1}{2}\log_2(32x^7)$

$\frac{1}{2}(\log_2 32+\log_2 x^7)$

$\frac{1}{2}(5+7\log_2 x)$

Exercise #8: If $\log 7=k$ then $\log(4900)$ can be written in terms of k as

(1) $2(k+1)$ (3) $2(k-3)$

(2) $2k-1$ (4) $2k+1$

$\log(4900)$

$\log(7\cdot 7\cdot 100)$

$\log(7^2\cdot 100)$

$\log 7^2+\log 100$

$2\log 7+2$

$2k+2$

$2(k+1)$

***Exercise* #9:** Consider the expression $\log_2\left(8^x\right)$.

(a) Using the third logarithm law (the Product Law), rewrite this as equivalent product and simplify.

$\log_2 8^x$

$x \log_2 8$

$x(3)$

$\boxed{3x}$

(b) Test the equivalency of these two expressions for $x = 0, 1,$ and 2.

$\log_2(8^0) = \log_2(1) = 0$ ✓

$3(0) = 0$ ✓

$\log_2(8^1) = \log_2(8) = 3$ ✓

$3(1) = 3$ ✓

$\log_2(8^2) = \log_2(64) = 6$ ✓

$3(2) = 6$ ✓

(c) Show that $\log_2\left(8^x\right) = 3x$ by rewriting 8 as 2^3.

$\log_2\left(2^3\right)^x = \log_2 2^{3x}$ ← The power we raise 2 to in order to get 2^{3x}

$\log_2 2^{3x} = 3x$